SIGHT REDUCTION FOR NAVIGATION

Table of Contents

SIGHT REDUCTION FOR NAVIGATION

I.
TIME

A. TIME ITSELF

Time is longitude. Geocentrically speaking the sun goes round the earth 360 degrees every twenty four hours, more or less. That is, spherically:

ANGLE		**TIME**		**DISTANCE** Equator	**DISTANCE** 30 North Lat
360°	equals	24 hours	equals	21,600 NM	14,400 NM
15°	equals	1 hour	equals	900	600
1°	equals	4 minutes	equals	60	40
15′	equals	1 minute	equals	15	10
1′	equals	4 seconds	equals	1	1,235 m
15″	equals	1 second	equals	1,235 m	309 m

Since Greenwich is 0° longitude (symbol lambda λ) and Dallas is λ 96° 48′ W, Dallas is 6 hours, 27 minutes, 12 seconds west of Greenwich in time. This is exact time from place to place so noon local mean time would be 18h27m12s GMT..

Once upon a time there was much ado about time zones, i.e, Central or Pacific, and about watch or chronometer error, but now since everyone has reference to GMT by his computer, watch or radio, GMT (recently officially called UTC) is the gold standard. You can even set it as your home page on the web by going to the USNO/NIST site http://www.time.gov. Local time and zone time can generally be disregarded.

B. TIME AND THE NOON SIGHT

1. LATITUDE BY MER PASS AT LAN

Local Apparent Noon (LAN) is an event, not a time system. You can determine LAN by looking at your GMT watch and timing successive observations around local noon or by calculation if you know the longitude. In Dallas, this would, by calculation, be at GMT 18h27m12s, but there is a catch. GMT means Greenwich Mean Time. It is mean time because it is averaged time. Depending on the time of year the actual Meridian Passage (Mer Pass) of the sun can be 16m25s earlier than the GMT or14m15s later than GMT depending upon “the obliquity of the ecliptic”.

These values are called the equation of time (Eq.T) and must be added or subtracted from your calculated GMT to arrive at the exact time of Mer Pass or LAN expressed in GMT. The values of the Eq. T can be found in the Nautical Almanac or on the Web at http://aa.usno.navy.mil/cgi-bin/aa_geocentric.pl. Be careful where you get the values for Eq.T: the sign + or - is an integral part of the value. Go to USNO for the US navigation values. In the US, the formula for apparent and mean time is: Apparent time equals Mean time minus Eq.T. Thus if Eq.T is positive, the time value is subtracted from the calculated GMT to arrive at LAN; if the Eq.T is negative, it is added to calculated GMT to arrive at LAN. The rule is backwards in more backward parts of the world like UK. Thus in Dallas, the calculated GMT for 0 Eq.T is GMT 18h 27m12s, but in February, Mer Pass is as late as 18h41m26s, and in November, it is as early as 18h 10m50s.

Graphically you can see both the Eq.T and declination (d) on an analemma- the strange figure eight on a globe. Different websites draw different analemmas and use different signs. However it is drawn, the May and November side is a +Eq.T and is subtracted from GMT to get LAN. The August and March side is -Eq. T and is added to GMT to determine the time of LAN.

The noon sunshot is acknowledged as the most reliable LOP of the day and is important to determining latitude because there is no sight reduction involved except addition and subtraction: 90° plus or minus declination minus Ho (observed altitude) equals latitude.

2. LONGITUDE BY MER PASS AT LAN, SUNRISE OR SUNSET

It is possible to determine longitude from the noon sunshot but you may well be inaccurate by a degree either way since the sun moves almost horizontally at noon and determining the exact time of LAN by a series of sunshots gives only a rough fix in about a three or four minute window. Assume that you have determined the GMT of LAN by observation. In order to use the time=longitude formula (one hour equals 15 degrees) you must also determine the GMT of LAN at Greenwich. Otherwise your measurement will be elongated or shorted by the Eq.T. So, if on July 8 the Eq. T is approximately -5m the LAN in Greenwich will be about 1205 GMT and the LAN in Dallas will also be 5 minutes later than expected, probably about 18h32. Remember that LAN is an event not a time system. To properly measure the time and distance on the earth’s surface remember that we have to measure the GMT of the LAN event at both ends of the calculation. You can also watch sunrise and sunset but the exact timing requires careful corrections.

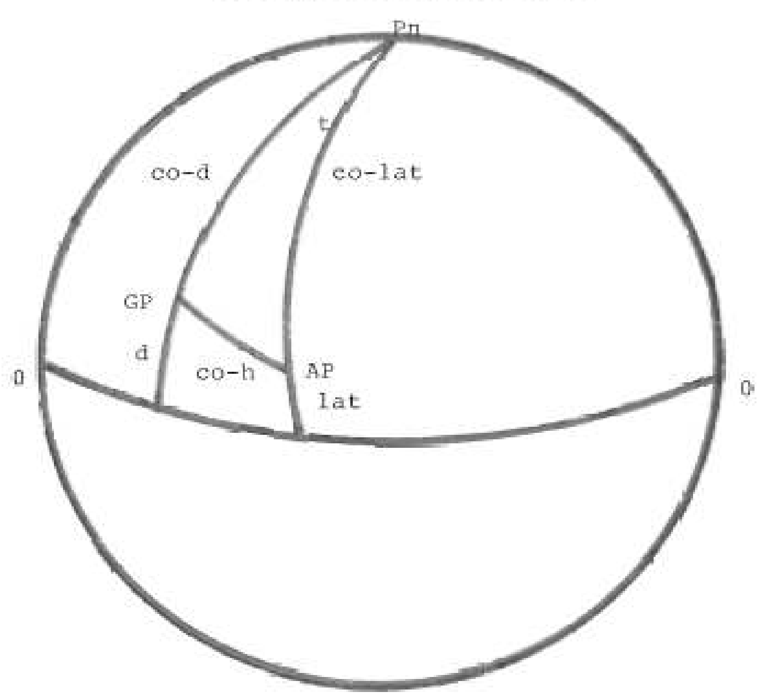
II.
THE NAVIGATIONAL TRIANGLE

Spherical trigonometry is rarely encountered in our times. The main practical use was air and sea navigation. The methods for sight reduction dealt with the "oblique" and "right" spherical triangles. The mathematical theory wasn't hard, but the tedious arithmetic was a killer. Logarithmic functions at least reduced the arithmetic to adding and subtracting. Today's hand calculator makes it easy.

A. NAVIGATIONAL TRIANGLE

Consider the navigational spherical triangle: The vertices are the pole (**P**), your assumed position (**AP**, also called **Z**), and the geographic position (**GP**, also called **M**) of the celestial body. You know two of the sides because from declination, you can determine its complementary polar distance or "**co-d**"; and, from estimated latitude, you can determine its complementary "**co-Lat**". What we are trying to learn is the third side between **AP** and **GP** since that side, known as "**co-H**", is the complement of **Hc**. The prerequisites are knowning LHA or t, declination or d of the celestial body, and assumed position AP. You will calculate LHA and learn d from the almanac; AP is your assumed position based on your dead reckoning. GHA Aries + SHA of the celestial body = LHA + Longitude.

THE NAVIGATIONAL TRIANGLE



B. OBLIQUE SPHERICAL TRIANGLE

Here are four formulas for solving the oblique spherical triangle without dividing it into two spherical right triangles. The first H.O. 9 formula is the root of all the rest. You can look up sin L and cos L, and sin d and cos d, together since the values usually appear across the page from each other in a book of trigonometric values. These are all variants of the law of cosines for sides.

1. H.O. 9 (1966 ed) page 528:

sin h = sin L sin d + cos L cos d cos t

which can be transformed into:

sin h = sin L sin d + sin (co-Lat) sin (co-d) cos t

2. Dutton (15th ed.) par. 2502:

sin H = sin L sin d + cos L cos d cos t

3. Nautical Almanac 2007 page 279:

Hc = $\sin^{-1}$ (S sin Lat + C cos Lat)

which transforms into:

sin Hc = (sin d sin L + cos d cos t cos L)

4. Hobbs (4th ed. 1998) page 411, uses the NAO notation which turns out to be:

sin Hc = (Sin L sin d + cos L cos d cos t)

These are all derived from the law of cosines. See Frank Ayers, Plane and Spherical Trigonometry (1st ed. 1954) page168.

The cosine of the side (side a) opposite a known angle A is:

cos a = cos b cos c + sin b sin c cos A.

If we re-label this formula in nautical terms, it is:

cos (co-H) = cos (co-d) cos (co-L) + sin (co-d) sin (co-L) cos t

Now transform:

sin H = sin d sin L + cos d cos L cos t

which is the basic formula used by H.O. 9. Obviously you know **d and t** from the Almanac, and have assumed **aL**, so you can do this with a hand calculator with ease.

An Example of solving the Oblique Spherical Triangle by Trigonometry:

Local time:	10:00 p.m., 31 Dec 2006
Place:	USNA, Lat 38 59; Long 76 29
UTC:	0300h 01 Jan 2007
Celestial body:	Betelguese
GHA	56° 29.9'
Dec	N 7° 24.6'
Hc	53° 39.8'
Zn	145.1°

The basics of the oblique spherical triangle "LTD":

L: aLat = **38° 59'**; co-lat = 51° 01'

T: Polar vertex angle, **t**, is determined by subtracting GHA of the body from longitude: 76° 29' minus 56° 30' = **19° 59'**

D: Declination = **7° 24.6'**; co-declination = 82° 35.4'

Using the H.O. 9 formula:

sin h = sin L sin d + cos L cos d cos t:

sin h = sin L (38° 59) sin d (7° 24.6) + cos L (38° 59) cos d (7° 24.6) cos t (19° 59)

sin h = (.62909) (.12908) + (.77715) (.99163) (.93979)

sin h = (.0812029) + (.7242446)

sin h = .8054475

H = 53° 39'

Co-h = 36° 21'

The azimuth of the celestial body can be determined by the law of sines.

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

We have just determined that t = 19° 59' and co-h = 36° 21' so the ratio has been established. Likewise we know that the side opposite vertex AP is the co-declination of 82° 35.4'. Thus the set up to be solved is:

$$\frac{\sin t}{\sin \text{co-h}} = \frac{\sin AP}{\sin \text{co-d}}$$

$\frac{\sin 19° 59}{\sin 36° 21}$	=	$\frac{\sin AP}{\sin 82° 35}$
$\frac{.34202}{.59272}$	=	$\frac{\sin AP}{.99163}$
sin AP	=	.57220
AP	=	34° 54'

This can't be true! Betelguese is in the southeast. Remember that sines go from 0 at 0° to 1 at 90° and back down to 0 at 180°. Thus sin AP at .57220 could be both 34° 54' or 145° 06'. Sin AP = Sin (180°-AP). In this case it is clear that the angle at **vertex AP** is **145° 06'**.

C. RIGHT SPHERICAL TRIANGLES

1. ARTHUR AGETON'S H.O. 211 SOLUTION. Ageton's solution to this spherical triangle is to divide it into two right spherical triangles by dropping a perpendicular (**H**) from **GP** to the opposite side (the longitude line including **AP** to **Pn**). This method was used by Arthur Ageton in 1931 and is the basis for H.O. 211. The formulas are straight-forward except they involve a lot of substitutions of co-functions of the complement of an angle for the function itself of that angle; i.e., sin 30° = cos 60°; sec d = csc (90°-d). In H.O. 211 Ageton uses log secants (the B column) and log cosecants (the A column). Remember co-d = 90-d; co-lat =90-lat. If h is the height of the celestial body, then co-h has to be the side of the triangle between GP and AP.

There are <u>three calculations</u> to determine Hc in the Ageton system:

1. The <u>first calculation</u> is determination of **R** which is the side between **M** and the right angle intersection of the perpendicular to side **Pn-AP** (or the extension of **Pn -AP**) at point **X**. We know two angles: the 90° degree angle at vertex **X**, and **t** or **LHA** at **Pn**; we also know side **co-d** as the complement of the declination of the heavenly body.

The formula would be: **csc R = csc t • csc (co-d)**

Ageton inverts csc (**co-d**) into sec **d**, and comes up with a formula:

csc R = csc t • sec d

Since Ageton uses logarithms, his formula is:

log csc R = log csc t + log sec d

2. The <u>second calculation</u> is determination of side **K** from the equator to **X** which is based on calculating **co-K** from **Pn** to **X**. **Co-K** is easily enough calculated because we know sides **co-d** and **R**. The fundamental formula is:

sec co-K = csc d / sec R

Ageton transforms this into: **csc k = csc d / sec R**

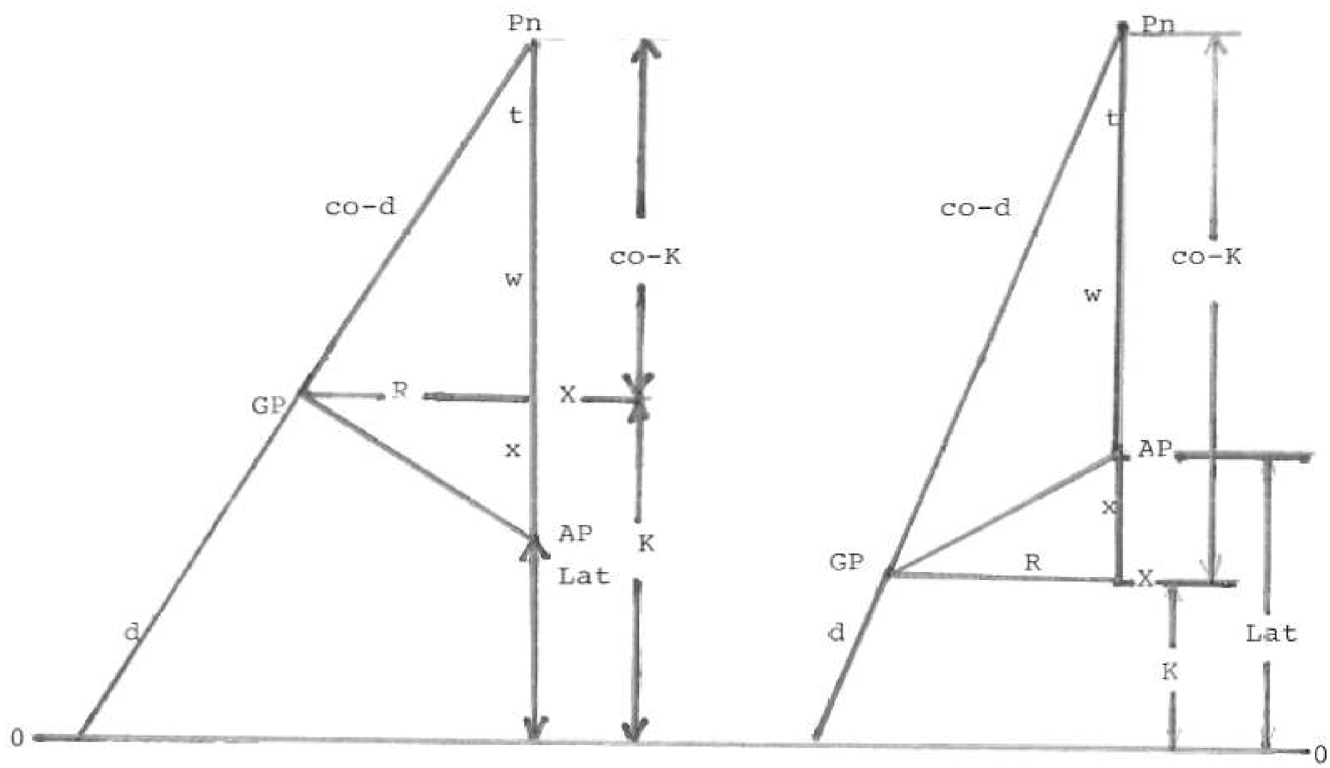
or, in logarithmic terms: **log csc K = log csc d - log sec R**

At this point subtract **aLat** from **K** if **aLat** is smaller and of the same sign or add **aLat** to **K** if they are of different signs. This produces a value which Ageton calls **K~L** and which is sometimes called **x**.

The Bayless Compact Sight Reduction Table (Modified H.O. 211) In 1980 Allan Bayless published a nine-page shortened version of the Ageton table of log secants and log cosecants. The entire booklet is 32 pages long and an easier presentation of Ageton's format and tables.

2. DREISONSTOK'S H.O. 208 SOLUTION Alternatively one can drop the perpendicular from the AP vertex to the co-d side as is done in the NAO Concise Sight Reduction Table or by Dreisonstok's H.O. 208.

.



D. COMPARISON OF AGETON, DREISONSTOK, AND H.O.9

1. Definitions. Both Dreisonstock and Ageton solve the oblique spherical triangle by dividing it into two right spherical triangles. Ageton drops a perpendicular from the GP of the celestial body to the P-AP line; Dreisonstock drops a perpendicular from the AP to the P-GP line. Some terminology is the same; some is different.

AGETON H.O. 211	**DREISONSTOK** H.O. 208	**BOWDITCH** H.O. 9 (1966)
LHA =t	LHA =t	LHA =t
P	P	Pn
Z = AP	Z = AP	Z= AP
M = GP	M = GP	M= GP
Co-L = colatitude	Co-L = colatitude	Co-L = colatitude
co-d = codeclination	co-d = codeclination	Co-d = codeclination
co-h = co-altitude side Z-M	co-h = co-altitude side Z-M	co-h = co-altitude side Z-M
R = the perpendicular from M intersecting at X on side P-Z; v; a	a = the perpendicular from Z intersecting at D on side P-M; v; r	v = the perpendicular from Z intersecting side P-M
X = 90 intersection on side Pn-Z	D= 90 intersection on side Pn-M	w
w = part side P-Z between P and X (90 -K); comparable to b	b = part side P-M between P and D; comparable to w	x
x = K minus Lat; Lat minus K; K~L	B = part side P-M between M and D	See figures 2110 and 2111.
K = equinoctial to X		
co-K = X to P; w		
L = equinoctial to Z		

2. Comparison of the Mathematics of Ageton and Dreisonstok

AGETON
H.O. 211

First Calculation:

Determination of right side **R** which is the side opposite t between M (GP) and the right angle at point X. We know two angles; the 90° degree angle at vertex X and t (or LHA) at P; we also know hypotenuse side co-d as the complement of declination d.
Remember a = R

Formula:

$$\csc R = \csc t \cdot \mathrm{Csc}\,(\text{co-}d)$$
$$\csc R = \csc t \cdot \sec d$$

$$\log \csc R = \log \csc t + \log \sec d$$

Second Calculation:

Determination of right side K from the equator to X which is based on calculating co-K from P to X. Co-K is easily enough calculated because we know sides co-d and R.

Formula:

$$\sec \text{co-}K = \csc d / \sec R$$
$$\csc K = \csc d / \sec R$$
$$\log \csc K = \log \csc d - \log \sec R$$

At this point subtract a Lat from K if aLat is smaller and of the same sign or add a Lat to K if they are of different signs. This produces as value which Ageton calls K~L and which is sometimes called x.

DREISONSTOK
H.O. 208

First Calculation:

Determination of right side **a** which is the side opposite t between Z (AP) and the right angle at point D. We know two angles: the 90° degree angle at point D and t (or LHA) at P; we also know hypotenuse side co-L as the complement of latitude L.
Remember a = R

Formula:
$$\sin a = \sin t \cdot \cos L$$
$$\sin a = \sin t \cdot \sin(\text{co-}L)$$
$$\csc a = \csc t \cdot \csc(\text{co-}L)$$
$$\csc a = \csc t \cdot \sec L$$

Second Calculation:

Determination of right side B

Formula:

$$\tan b = \cot L \cos t$$

$$B = 90 - d - b$$

$$B = 90 - (d+b)$$

Third Calculation.	**Third Calculation**:
Determination of **h** (altitude) or **co-h** (hypotenuse side between AP and GP) when right sides R and K~L are known. Formula: **sec co-h (side) = sec R • Sec x** **csc h (alt) = sec R • sec x** remember (a = R) and (x = B)	Determination of **h** (altitude) or **co-h** (hypotenuse side between Z and M) when right sides a and B are known. 90° = B + b +d Formula: **sin h (alt) = cos a • cos B** **sin h (alt) = cos a • sin (d+b)** **cos co-h (side) = cos a • cos B** **sin h = cos a • sin B** **csc h (alt) = sec a • csc (d +b)** **csc h(alt) = sec a • sec B** remember (a = R) and (x = B)

III.
THE GREAT CIRCLE TRACK

Exactly the same formulas for an oblique spherical triangle which apply to determining Hc apply to determining the mileage of the great circle track between two ports (after all, what is co-H except the great circle between AP and GP). The polar angle t is known; the two adjacent sides, colatitudes are calculated by subtracting the latitude from 90°.

St. John’s Newfoundland is at 47° 34' N 52° 42' W; its co-latitude is 42° 26'.

Kinsale Light Old Head is at 51° 36' N 8° 32' W; its co-latitude is 38° 28'.

The polar angle (known as t) between these two longitudes is 44° 10', cos A = .71732.

	Co-Lat	Cos	Sin
St. Johns	42° 26'	.73806	.67473
Kinsale	38° 28'	.78297	.62092

Using the cosine formula for two sides and an included angle, we have:

cos a = cos b cos c + sin b sin c cos A

cos a = (.73806) (.78297) + (.67473) (.62206) (.71732)

cos a = (.577878) + (.301075)

cos a = 0.87895

a = 28° 29' x 60 nm/degree

a = 1707 nm

You might initially think that it would be easier to work out a right triangle with longitude and latitude lines but recall that, although all longitudes are great circles, the only latitude line which is a great circle is the equator.

With these formulas it is easy to determine the course at beginning and at end, but the proper course throughout the voyage is always changing. Use an AP for the beginning of the voyage and recalculate.

IV.
USEFUL REFERENCES

The website http://aa.usno.navy.mil is the Astronomical Applications Department of the United States Naval Observatory. It carries on the web the calculations that one would make from the Nautical Almanac and H.O. 229 (which has replaced H.O. 214). The Eq.T for any particular time is at http://aa.usno.navy.mil/cgi-bin/aa_geocentric.pl, and a complete set of GHA, declination, and Hc for selected bodies at the chosen AP is at http://mach.navy.mil/cgi-bin/aa_flamenav.pl.

Spherical trigonometry is treated in Frank Ayers, Jr., Plane and Spherical Trigonometry (Schaums's Outline Series, 1st ed. 1954), and in Kells, Kern & Bland, Spherical Trigonometry with Naval and Military Applications (USNA, 1942).